Titolo del corso: Fine properties of functions of bounded variation and sets of finite perimeter

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Ore frontali di lezione: 16 h

Periodo di lezione: novembre-dicembre 2026

Settore/i disciplinare del corso: Analisi Matematica

Tipologia di corso: Base

Modalità di verifica dell'apprendimento: seminario su un argomento da approfondire

Abstract del corso:

A function of bounded variation (BV) is a summable function whose distributional gradient is a finite vector valued Radon measure. Hence, the space of BV functions represents a natural extension of the Sobolev space $W^{1,1}$, and, due to its crucial weak compactness property, it plays a fundamental role in Calculus of Variations. In particular, there are measurable sets whose characteristic functions belong to the space BV: these are called sets of finite perimeter, and they generalize the idea of open sets with smooth boundary. Despite being quite irregular, BV functions enjoy interesting fine properties, such as the existence of precise representatives up to codimension one Hausdorff negligible sets. The aim of this course is to present the fundaments of the theory of BV function. We will start by providing basic notions of measure theory (especially concerning Radon and Hausdorff measures), and then we will prove many properties of functions of bounded variation, with a particular focus on De Giorgi's blow-up and structure theorems for sets of finite perimeter. Afterwards, we will exploit the coarea formula to prove the approximate continuity of a BV function outside its jump set and the absolute continuity property of its gradient. Finally, we will treat the decomposition of the gradient of a BV function and Federer-Vol'pert theorem, leading to Vol'pert's product rule and integration by parts formulas for BV functions on sets of finite perimeter.

Programma del corso:

Notions of measure theory. Radon measures, Riesz Representation Theorem and weak* convergence.

Basic properties of the Hausdorff measure. Differentiation of measures, area and coarea formulas, integration by parts formula on regular open sets.

Functions of bounded variation. Basic properties and relation with Sobolev functions. Anzellotti-Giaquinta approximation theorem, embeddings and compactness of the space BV.

Sets of finite perimeter. Coarea formula for BV functions. Reduced boundary. Isoperimetric inequalities. Localization and density estimates. Measure theoretic boundary and blow-ups. De Giorgi's blow-up theorem. Rectifiability and De Giorgi's structure theorem.

BV functions in one dimension.

Continuity and jump sets of a BV function. Precise representative. Gradient decomposition and Federer-Vol’pert theorem. Vol’pert’s product rule and integration by parts formulas on sets of finite perimeter.